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Technical Notes

Initial Perturbation Amplitude of Liquid Sheets Produced by Jet-Impingement Nozzles

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Nomenclature

d	=	diameter
f	=	breakup parameter
h	=	sheet thickness
K	=	sheet parameter
Oh	=	Ohnserge number
R	=	jet radius
Re	=	Reynolds number
r	=	distance from impingement point to plate tip
U	=	velocity
We	=	Weber number
η	=	wave amplitude
μ	=	viscosity
ρ	=	density
σ	=	surface tension

Subscripts

D	=	droplet
g	=	gas
j	=	jet
L	=	ligament
l	=	liquid
m	=	mean
p	=	plate

I. Introduction

THE liquid-film atomization nozzles have numerous applications, such as in gas turbines, liquid-fueled rocket engines, automobile engines, and recovery boilers in the pulp and paper industry. One method of film formation is by impinging a liquid jet on a solid surface. This method has several advantages, such as low injection-pressure loss and high controllability of the generated liquid film [1]. In this type of nozzle, a jet of fluid impinges on a splash plate and spreads out radially while thinning. The formed liquid sheet breaks into small droplets under the influence of the surrounding medium, turbulence, etc.

There have been numerous studies on the sheet breakup process [2–12], which have identified different modes of disintegrations, droplet size distribution, etc. The sheet may break due to aerodynamic wave instability, laminar edge instability, laminar tearing, turbulent edge instability, or turbulent tearing and perforation. The sheet instability is described based on an instability theory using both linear and nonlinear theories. The linear instability theory considers the growth of infinitesimal disturbances due to aerodynamic stresses on the surface of a liquid sheet due to disturbances originated in the pipe and at the injection time. The wave with the maximum growth rate is assumed to cause the breakup of the sheet. Various relations have been developed for the maximum growth rate of such disturbances. Essentially, all the proposed models provide a perturbation amplitude in the following form:

$$\eta = \eta_0 e^{\alpha_{\max} t} \tag{1}$$

where $\alpha_{\max}$ is the maximum growth rate of the instability wave. The growth rates obtained based on the linear theory are independent of the initial disturbance amplitude. However, to find the breakup time and breakup length, we need to use Eq. (1), which requires η_0, the initial disturbance amplitude, which is generally not available. To overcome this problem, the experimentally measured breakup time or breakup length are used to estimate a value for the initial disturbance amplitude, which in fact represents an empirical parameter adjusting the theoretical model to match the experimental values. This Note examines the validity of the commonly used parameters for the initial amplitude and provides a new procedure to determine the initial amplitude with improved accuracy and universality.

II. Initial Wave Amplitude

The breakup time of a liquid sheet can be predicted using the growth rate of a perturbation wave and its initial amplitude. Inversely, the breakup time can also be used to infer initial perturbation amplitude. The breakup time of a liquid sheet can be found based on the assumption that breakup occurs when the disturbance amplitude becomes the same as the sheet thickness, $\eta = h$. The result is

$$t = \frac{L}{U} = \frac{1}{\alpha_{\max}} \ln\left(\frac{h}{\eta_0}\right) = \frac{f}{\alpha_{\max}} \tag{2}$$

where f is referred to as the breakup parameter and it is determined using experimentally found breakup times. The liquid ligament and droplet sizes can then be determined using the wavelength responsible for the breakup. Dombrowski and Johns [12] assumed that the liquid sheet breaks up into cylindrical ligaments, which in turn become unstable and break into droplets. They provided the following equation for the ligament size produced by the instability and breakup of a two-dimensional attenuating liquid sheet:

$$d_L = 2\left(\frac{4}{3f}\right)^{1/3}\left(\frac{k^2\sigma^2}{\rho_g \rho_L U_s^2}\right)^{1/6}\left[1 + 2.6\mu_l \sqrt[3]{\left(\frac{k\rho_g^4 U_s^8}{6f\rho_l^2\sigma^5}\right)}\right]^{1/5} \tag{3}$$

where, in the case of jet-impingement nozzles, k is the thickness parameter at the plate tip, defined as

$$k = h_p r_p \tag{4}$$

Dombrowski and Johns [12] successfully applied their model to predict droplet size produced by a fan spray. Inamura et al. [1] used

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the same method to predict the droplet size for splash-plate nozzles, which compared well with those of experiments.

The droplet size can then be calculated using the ligament diameter. Dombrowski and Johns [12] gave the following relation for the droplet size:

$$d_D = 1.882 d_L [1 + 30h]^{1/6} \tag{5}$$

where Oh is the Ohnserge number, defined as

$$Oh = \frac{\mu_l}{\sqrt[2]{\rho_l \sigma d_L}} \tag{6}$$

As noted, ligament and droplet sizes are dependent on the initial perturbation amplitude or its ratio to the final wave amplitude, which is represented by the breakup parameter f. The pioneering investigation on this parameter is by Weber [13], who obtained a value of 12 using jet breakup experiments:

$$f = \ell n \left(\frac{\eta}{\eta_0} \right) = 12 \tag{7}$$

Other researchers have reported different but similar numbers. Grant and Middleman [14] proposed a value of 13.4 for a jet of glycerol/water solution and Kroesser and Middleman [15] proposed a value of 11 for viscous Newtonian liquid with Ohnserge numbers between 0.28 and 1.03. Research by Fraser and Eisenklam [4] suggested that the value obtained by Weber [13] can be used in sheet breakup models. They tested a fan-spray sheet and confirmed that for a given nozzle, the breakup parameter remains constant. Dombrowski and Johns [12] as well as Inamura et al. [1] used the same quantity in their analysis and found poor predictions of droplet sizes as compared with those of the experiments.

Therefore, Dombrowski and Johns [12] suggested matching the predicted droplet size d_D with the measured droplet size d_m using a correction factor of 0.676:

$$d_m = 0.676 d_D \tag{8}$$

Because of the differences in the experimental condition, such as injection nozzles and fluid properties, different correction factors are reported by different researchers. Using Dombrowski and Johns's [12] correction factor of 0.676 in Eq. (5) and (6), we get

$$d_D = 1.186 d_L [1 + 30h]^{1/6} \tag{9}$$

Similarly, using Inamura et al.'s [1] correction factor, predicted droplet size becomes

$$d_D = 0.6 d_L [1 + 30h]^{1/6} \tag{10}$$

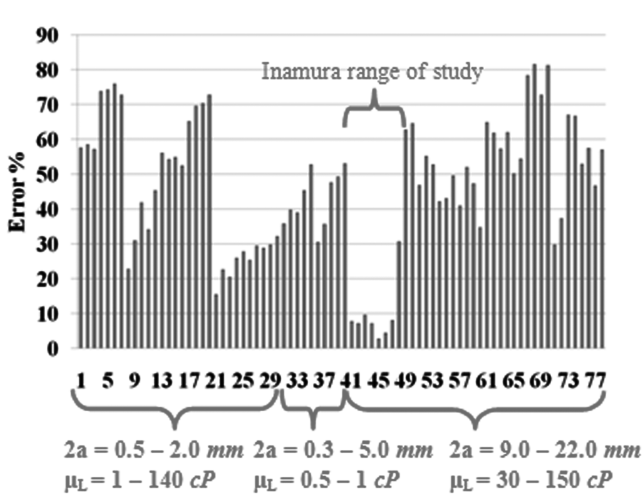


Fig. 1 Prediction error for Inamura et al.'s [1] correlation compared with 67 different experimental data sets.

Table 1 Range of physical properties for breakup-point experiments

Properties	Values
Nozzle diameter	0.3–2.0 mm
Injection velocity	5–65 m/s
Density	700–1400 kg/m^3
Viscosity	0.7–140 mPa · s
Surface tension	20–75 mN/m

Ahmed et al. [16] also conducted a detailed spray sizing on a splash-plate atomizer. Using their correction factor, the predicted size becomes

$$d_D = 0.93 d_L [1 + 30h]^{1/6} \tag{11}$$

Dombrowski and Johns's [12] experiments are on fan sprays, and Inamura et al.'s [1] and Ahmed et al.'s experiments [16] are on splash-plate atomizers. Fan-spray and splash-plate atomizers are in the same category of nozzles; however, different correction factors are proposed. Each of these equations is valid in a specific range of operation, and they result in large errors if used out of their intended range. To show this, Inamura et al.'s [1] correlation is applied to several different experimental data, including cases that are out of the recommended range of operation. Figure 1 shows the error for the calculated droplet diameter based on the Inamura et al. relation with respect to experimental results. Errors for the range that the equation is developed for are less than 10%, but for other ranges, the minimum error is 20%, with a maximum error as high as 82%.

It is clear from Fig. 1 that Eq. (10) gives satisfying results only in a narrow range that it is developed for and it is not accurate in other conditions. Similar results are obtained for Dombrowski and Johns's [12] model. The existence of a wide range of correction factors indicates that f may not be a constant parameter and it is dependent on liquid properties and the flow conditions. To resolve this problem, a new method of calculating the breakup parameter is introduced here. We have used a wide range of experimental data to obtain a correlation, not a constant, for the breakup parameter. Three different experimental data sets are used to develop a correlation for the breakup parameter. These include data by Ahmed et al. [16], Kankkunen and Nieminen [17], and Inamura et al. [1]. These experiments are selected because they cover a wide range of injection and fluid conditions, which makes the final correlation applicable to a wide range of operating conditions. The range of physical properties is shown in Table 1.

The droplet size data from these experiments are used in Eq. (5) along with Eq. (3) to obtain a breakup parameter for each of the cases; then a correlation for f as a function of fluid physical properties, nozzle diameter, and injection velocity in a nondimensional form is obtained. The final correlation is

$$f = Re^{0.07} We^{0.37} \tag{12}$$

where Re is the Reynolds number, and We is the Weber number based on the injection velocity, nozzle diameter, and fluid properties.

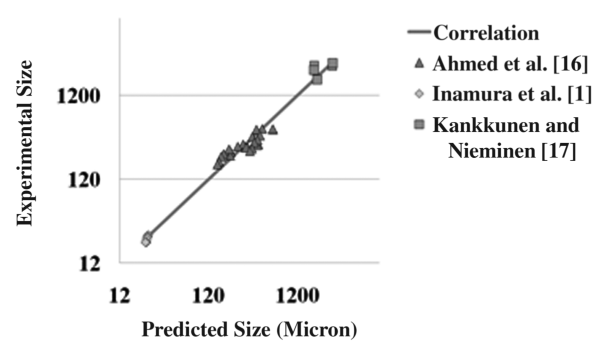
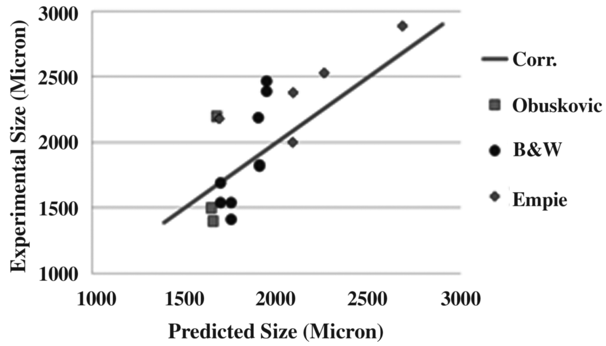
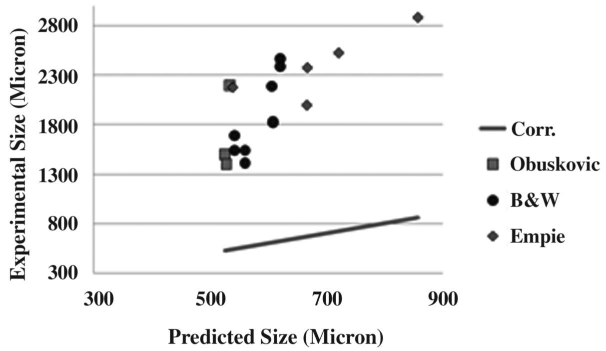


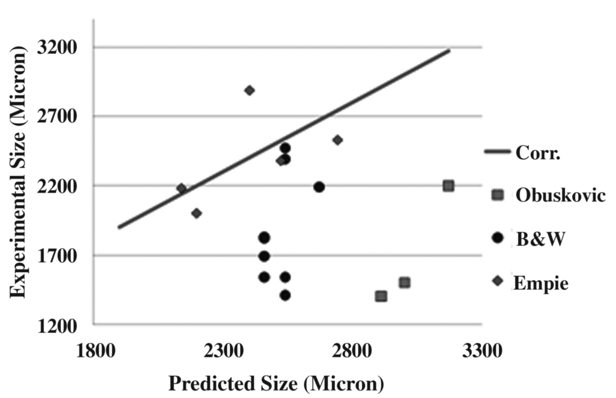
Fig. 2 Data used for correlating breakup parameter formula.



a) Equation (12)



b) Inamura et al. [1] correlation



c) Adams [21] correlation

Fig. 3 Comparison between predicted and experimental droplet sizes using three different correlations [Obuskovic and Adams [18], Babcock and Wilcox (B&W) as reported in [19], and Empie et al. [20].

III. Validation and Comparison

Figure 2 shows the accuracy of the correlation as compared with the data used to develop it. The axes are in logarithmic scale, as the experiments are from three different ranges of droplet sizes. Figure 2 indicates that Eq. (12) is a good correlation for the three data sets used. To validate the applicability of Eq. (12) to a wide range of experimental data, it is compared with three other experimental data sets. Figure 3 shows the outcome of such a comparison.

Figure 3a is the predicted droplet size from our newly proposed model [using Eq. (12)] and Fig. 3b compares Inamura et al.'s [1] correlation [Eq. (10)] with the same set of experiments used in Fig. 3a. Their correlation does not accurately predict the data over the range of conditions used. The error for their correlation increases toward the larger droplet sizes and it can be as high as 100% error.

Figure 3c illustrates the droplet size predictions by Adams's [21] correlation. This correlation is widely used in the pulp and paper industry to predict the droplet sizes generated by large-scale splash-plate nozzles. Although Adams's [21] correlation is more accurate than Inamura et al.'s [1], it is still less accurate than our newly proposed procedure.

Table 2 summarizes the difference between three different droplet size predictions using Inamura et al. [1], Adams [21], and the present model with those of experiments. The least accurate method is Inamura et al.'s [1] correlation, which has an average error of 68%; however, it gives accurate results for small nozzles and fluids with viscosity near that of water. Adams's correlation [21] which is used in an industrial scale, gives more accurate results compared with those of Inamura et al. [1]; however, it still results in an average error of 37%. The new model has the best accuracy, with a maximum error of 24% and average error of 12%. Comparing average errors for these two methods (constant parameter versus variable parameter) reveals that the new method of calculating the breakup parameter results in noticeably improved accuracy compared with those with a constant parameter. Its other advantage is that it is applicable to a wide range of operating conditions, whereas none of the previous correlations could be applied.

Table 2 Comparison of errors with respect to experimental results between different models to predict droplet size distribution

	Min error, %	Max error, %	Average, %
Inamura et al. [1] (constant parameter)	60	76	68
Adams [21] (constant parameter)	1.9	85	37
New correlation (variable parameter)	0.5	24	12

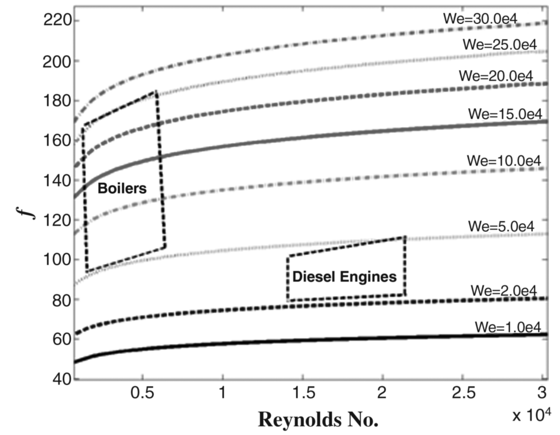


Fig. 4 Breakup parameter for different applications of sheet breakup.

Figure 4 shows the breakup parameters calculated from Eq. (12) for different Weber and Reynolds numbers. The ranges for two important applications of jet-impingement nozzles are also shown (boilers and direct-injection diesel engines). The breakup parameter increases by increasing both the Weber and the Reynolds numbers, although this increase is very gradual. For Reynolds numbers greater than 30,000, the wave instability breakup is not applicable and the breakup is mostly driven by turbulent effects such as perforation and tearing.

Note that the breakup parameters obtained using the present correlation are, in some cases, 1 order of magnitude larger than previously reported values. More investigations reveal that by changing the breakup parameter by 1 order of magnitude, the droplet size changes only by a factor of 2. Therefore, the droplet size is insensitive to large changes in breakup parameter, and to have a wide range of droplet sizes, a variable and large breakup parameter should be used.

IV. Conclusions

A breakup parameter is introduced to predict the droplet sizes produced by splash plate and impinging-jet atomizers. This parameter is obtained based on using a range of experimental data. The newly developed breakup parameter is

$$f = Re^{0.07} We^{0.37}$$

which is a function of Reynolds and Weber numbers. This flow dependency is also confirmed by Grant and Middleman [22] based on their experimental findings. Increasing the Reynolds number (less viscous fluid), the sheet becomes more unstable; therefore, a smaller initial perturbation can make the sheet break. It means that a larger breakup parameter will be obtained. By increasing the Weber number (decreasing surface tension), a smaller perturbation is able to perturb the sheet and cause the breakup. This is in contrast to the previously used constant breakup parameter. This means that a dependent breakup parameter will be obtained.

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